Hi and welcome,

This is a brief sample to provide you with an idea of what my IMT Exam study notes look like.

The style is to provide key content coverage followed by working examples.

If you have written the exam before, you can use my materials to get back up to speed very quickly.

If you are a first time writer, you can use my materials to help focus your study efforts and build your knowledge base.

If you have any questions, please do not hesitate to reach out to me by email.

All the best,

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Chapter 9
Analysis of Debt Securities I: Valuation, Term Structure, and Pricing

Introduction

IAs must understand the potential risks and benefits of investing in debt securities to be able to make the right recommendations. This chapter discusses the valuation of debt securities, including strip coupons, fixed-coupon bonds, and money market securities.

How to Value Debt Securities

The Time Value of Money

The time value of money is a very important concept. For example, you have $50,000 in cash right now. This is the present value of your money. Now, assume that you deposit the money in a bank account with an interest rate of 5% paid per year (5% * $50,000 = $2,500 interest in one year). Consequently, after one year, you will have $52,500, which is the future value of your money. If you keep the $52,500 in the account for one more year, another 5% interest payment is received ($2,625), which means that the future value of your money two years from now is $55,125. The interest in the second year was $125 greater because the interest was not only earned on the initial amount of $50,000 but also on the interest of $2,500 received in the first year. This concept is known as compound interest (it is compounding each year).

Equation 1 - Future Value

$$FV = PV * (1 + i)^n$$

Where:
PV = Present value
FV = Future value
i = Interest rate per period, usually shown as I/Y on a financial calculator)
n = Number of compounding periods

Supposed that the amount of $50,000 is left in the account for 5 years, and each year you receive 5% interest. The future value five years from now would be calculated as:

$$FV = \$50{,}000 * (1 + 0.05)^5 = \$50{,}000 * 1.2762 = \$63{,}814$$

The reverse is also true. So, for instance, if you want to obtain $70,000 in 5 years from now, you can calculate how much you need to invest today (present value) to obtain that return.

Equation 2 - Present Value

$$PV = \frac{FV}{(1+i)^n}$$

$$PV = \frac{\$70{,}000}{(1+0.05)^5} = \$54{,}850$$

So, if you want to generate $70,000 in the next five years, you need to invest $54,850 today.

All debt securities are priced using this concept, which states that the price of any debt security is the present value of its expected cash flow discounted at an appropriate interest rate. The appropriate interest rate for a debt security is known as the yield or yield to maturity. However, equation 2 is not applicable to most debt securities because the present value of some debt security must also consider the number and timing of expected cash flows and other aspects that vary between debt securities.

Strip Coupons

To calculate the yield/price of a strip coupon or residual, you need two variables: the price (when finding out the yield) or the yield (when finding out the price), and the number of days to maturity. Since most bonds in Canada pay interest twice a year, the yield to maturity on bonds is actually a semi-annual yield.

To make the yield on strip coupons and residuals directly comparable to the yield on other bonds, strip coupon yields are often quoted with equivalent semi-annual yields and annual yields. In this chapter, "semi-annual yield" actually means "two times the semi-annual yield" (annualized yield). Most firms provide both yields to IAs through their order entry system.

Equation 3 - Price of a Strip Coupon/Residual (Annual Yield)

$$Price = \frac{100}{(1+\ y_A)^{\frac{n}{365}}}$$

Where:
y_A = The equivalent annual yield
n = The number of days to maturity measured from the settlement date

If the settlement date happens to coincide with the maturity date, the (n/365) expression can be replaced with the number of years to maturity. Otherwise, the exact number of days is used.

Example:
A Province B residual maturing on June 23, 20X5, is offered with an equivalent annual yield of 5% for settlement on June 30, 20X2. There are 1,102 days from settlement to maturity. The price of this residual is calculated as follows:

$$Price = \frac{100}{(1+\ 0.05)^{\frac{1,102}{365}}} = \frac{100}{1.1587} = \$86.30$$

This means that the price of the Province B residual per $100 face value is $86.30.

Equation 4 - Price of a Strip Coupon or Residual (Semi-Annual Yield)

$$Price = \frac{100}{(1+\ y_A/2)^{\frac{2*n}{365}}}$$

If the settlement date happens to coincide with the maturity date, the (2 × *n*/365) expression can be replaced with (2 × the number of years to maturity). Otherwise, the exact number of days is used.

Example:
If a Province B residual is offered with an equivalent semi-annual yield of 5%:

$$Price = \frac{100}{(1+\ 0.05/2)^{\frac{2*1,102}{365}}} = \frac{100}{1.1607} = \$86.15$$

Given the price and the number of days to maturity, the following equation can be used to calculate the equivalent *annual* yield of a strip coupon or residual:

Equation 5 - Equivalent Annual Yield

$$Equivalent\ Annual\ Yield = \left(\frac{100}{Price}\right)^{\frac{365}{n}} - 1$$

If the Province B residual is offered at a price of $86.30, its equivalent annual yield is calculated as follows:

$$Equivalent\ Annual\ Yield = \left(\frac{100}{\$86.30}\right)^{\frac{365}{1,102}} - 1 = 0.0500 = 5\%$$

Equation 6 - Equivalent Semi-Annual Yield

$$Equivalent\ Semi - Annual\ Yield = \left[\left(\frac{100}{Price}\right)^{\frac{365}{2*n}} - 1\right] * 2$$

If the Province B residual is offered at a price of $86.30, its equivalent semi-annual yield is calculated as follows:

$$Equivalent\ Semi - Annual\ Yield = \left[\left(\frac{100}{\$86.30}\right)^{\frac{365}{2*1,102}} - 1\right] * 2 = 0.0493 = 4.93\%$$

Bonds

For a bond with a term to maturity of greater than one year, calculating its price given the yield requires one present value calculation for each expected payment. This section shows how to determine the price only when the settlement date falls on a coupon payment date.

Example:

Consider an annual-pay bond with a 5% coupon and 2 years until maturity, and an annual yield of 6%. This means that the bond will pay 2 coupons of $5 each and return the $100 principal at maturity (after 2 years).

To find out the price of the bond today, you need to calculate the present value of each coupon payment and the final principal repayment and add them together. Knowing that the last coupon will be paid on the same day the principal is repaid, the price of the bond is calculated as follows:

$$Price = \frac{5}{(1+0.06)^1} + \frac{100+5}{(1+0.06)^2} = \frac{5}{1.06} + \frac{105}{1.1236} = 4.7169 + 93.449 = \$98.17$$

However, most Canadian bonds pay the coupon twice per year. In this case, the following calculation is applied:

Consider an annual-pay bond with a 5% coupon and 2 years until maturity, and a semi-annual yield of 6%. This means that the bond will pay 2 coupons of $5 each and return the $100 principal at maturity (after 2 years). However, since the coupon is paid twice, this means that the investor receives $2.50 twice per year, while the semi-annual yield is 3% (the interest rate used to discount the cash flow). Since payments are made twice per year, there will be four payments for the 2 years.

$$Price = \frac{2.5}{(1+0.03)^1} + \frac{2.5}{(1+0.03)^2} + \frac{2.5}{(1+0.03)^3} + \frac{100+2.5}{(1+0.03)^4}$$

$$Price = \frac{2.5}{1.03} + \frac{2.5}{1.0609} + \frac{2.5}{1.0927} + \frac{100+2.5}{1.1255}$$

$$Price = 2.427 + 2.356 + 2.2879 + 91.07 = \$98.14$$